and one only, which makes $\gamma_B/\gamma_Y = \lambda_B/\lambda_Y = 0{\cdot}73$, viz., when $r = 40^\circ\ 29'$ ($45'$ less than the angle of total reflection for the yellow and $24'$ less than for the blue).

At the position so defined there will be a band with borders only faintly coloured. On either side of this band the order in which the colours appear will be reversed.

On the Efficiency of Muscular Work.

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(Communicated by Prof. Leonard Hill, F.R.S. Received January 18, 1918.)

In a paper communicated to the Royal Society in 1913,* Prof. J. S. Macdonald published a series of observations upon the heat production of persons performing certain known quantities of work upon a bicycle ergometer. In that paper, and again in a more recent publication,† Prof. Macdonald has outlined certain methods of interpreting his results, which are of much importance; these I shall discuss in the latter half of this communication, but, before doing so, it will be interesting to examine some purely numerical questions to which Macdonald's paper gave rise.

In a note on Macdonald's earlier paper, Messrs. Glazebrook and Dye‡ have published a formula descriptive of Macdonald's numerical results. This formula is

$$H = a + bM + \frac{W}{\alpha + \beta M}, \tag{1}$$

where H = heat production in calories, M = body mass in kilogrammes, W = work equivalent in calories, a, b, α, and β are constants.

The values of the constants, which were obtained by a graphical process led in the particular case to the equation

$$H = -138 + 4{\cdot}5M + \frac{W}{0{\cdot}08 + 0{\cdot}003M}, \tag{2}$$

and this equation was found to provide values in very fair agreement with the observed results.

It will be noticed that, when the body weight is constant, the heat production is, by Glazebrook and Dye's formula, a linear function of the external work performed. For a constant performance of work, the change of heat production with changing mass is described by a hyperbola, one asymptote of which is parallel to the axis of H (for W = 0, the equation is that of the asymptotes), and, in consequence of the position of the principal axes with reference to the axes of H and M, the minimum of H moves to the right with increasing values of W.

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* 'Roy. Soc. Proc.,' B, vol. 87, p. 96 (1914).

† *Ibid.*, vol. 89, p. 394 (1917).

‡ *Ibid.*, vol. 87, p. 311 (1914).

In effect, we have

$$\frac{dH}{dM} = b - \frac{W\beta}{(\alpha+\beta M)^2}, \tag{3}$$

$$\frac{d^2H}{dM^2} = \frac{2\beta^2 W}{(\alpha+\beta M)^3}, \tag{4}$$

(4) is always positive and from (3) the value of M for a minimum is

$$M = -\alpha/\beta \pm \sqrt{(W/b\beta)}.$$

This decreasing disadvantage, or, rather, increasing advantage, of mass with increase of work is in agreement with the remarks of Prof. Macdonald,* and, *pro tanto*, is an argument in favour of the form of Glazebrook and Dye's expression. But that the formula itself is no more than an interpolation formula is manifest, since it gives negative values of H for small values of M when W = 0, while, even within practicable ranges, it leads to the paradoxical result that the absolute heat production associated with the performance of 56 thermal units of work in a mass of 20–30 kgrm. is not less than that of a mass between 50 and 60 kgrm. I thought, therefore, that it would be of interest to determine whether a linear interpolation formula might not reproduce the experimental results with an accuracy comparable with that achieved by Glazebrook and Dye's second degree expression. Taking the data on p. 313 of Glazebrook and Dye's paper, I computed the various constants of a first degree multiple regression equation; these are set out in Table I, and the deduced equation is

$$H = 24{\cdot}900 + 3{\cdot}940\,W + 1{\cdot}755\,M. \tag{5}$$

Testing this against the observations, I obtain Table II, which is a reproduction of Glazebrook and Dye's Table III, with the addition of the values computed from the above regression equation. It is easy to see that the linear expression is not inferior to Glazebrook and Dye's expression in its power to reproduce the experimental results, although it has one less constant (actually, the mean square of error is rather smaller in the present case, but this is largely because of the failure of Glazebrook and Dye's formula in connection with the individual weighing 43·7 kgrm., and, in any event, the test is a crude one, so that no claim is made that the regression formula is the better of the two). The regression equation does not lead to any paradoxical negative values of heat production, but that it also is only an interpolation formula is seen in the circumstance that the computed heat production at rest exceeds the experimentally determined values of the heat production in sedentary occupations, and, *a fortiori*, those of persons resting in bed.

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* *Op. cit.*, p. 111.

The result, however, suggested the desirability of examining in the same way other data.

Table I.—Constants Deduced from Data of Glazebrook and Dye's Table II (*op. cit.*, p. 313).

	Heat production.	Work.	Mass.
Mean	260·042	34·042	57·542
Standard deviation ...	64·515	16·016	6·452

$r_{HW} = 0\cdot9735$ $\quad {}_{M}r_{HW} = 0\cdot9889$ $\quad {}_{WM}\Sigma_{H} = 9\cdot478$

$r_{HM} = 0\cdot1493$ $\quad {}_{W}r_{HM} = 0\cdot7668$ $\quad {}_{HM}\Sigma_{W} = 2\cdot379$

$r_{WM} = -0\cdot0269$ $\quad {}_{H}r_{WM} = -0\cdot7608$ $\quad {}_{HW}\Sigma_{M} = 4\cdot140$

For this purpose I selected the observations of Amar.* This investigator also made use of a stationary bicycle as an instrument for measuring work performance. The subjects of the experiment were Algerians accustomed to heavy manual labour, and the period of trial several hours. The determinations of heat production were much cruder than those of Macdonald, and depend upon analyses of the ingredients of a diet sufficient to maintain the subject in equilibrium (it would seem that constancy of weight was all that was attempted to be maintained, separate balance sheets of the factors involved not being, from the nature of the experiments, capable of preparation) or upon the oxygen use. The heat productions determined by the two methods did not agree at all closely, and I have followed the author in paying special attention to the heat values of the equilibrium diets. Table III reproduces the observations, the amounts of work having been converted from kilogramme-metres to thermal units.

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* 'Le Rendement de la Machine humaine,' Jules Amar, Paris, 1910.

Table II.—Comparison of Observed and Calculated Heat Production.

Work per-hour in calories	0			13			19			43			56		
Body mass in kgrm.	Heat production.			Heat production.			Heat production.			Heat production.			Heat production.		
	Obs.	Calc. I.	Calc. II.	Obs.	Calc. I.	Calc. II.	Obs.	Calc. I.	Calc. II.	Obs.	Calc. I.	Calc. II.	Obs.	Calc. I.	Calc. II.
43·7	—	59	102	—	—	—	177	149	176	279	263	271	346	319	322
54·6	—	108	121	160	161	172	193	186	196	280	284	290	335	338	341
55·7	—	113	123	169	166	174	—	—	—	285	287	292	—	—	—
58·8	—	127	128	181	178	179	—	—	—	298	295	298	—	—	—
60·5	—	134	131	—	—	—	212	206	206	317	299	300	347	348	352
61·9	—	141	134	186	190	185	216	212	208	306	303	303	348	352	354
66·7	—	162	142	209	208	193	—	—	—	324	315	311	352	362	363

Table III.—Observations of Amar.

Weight.	Work.	Heat production.	Weight.	Work.	Heat production.
kgrm.	calories.	calories.	kgrm.	calories.	calories.
76·2	156·8	3398	64·8	137·5	3020
71·3	114·1	2988	60·2	129·7	2812
69·6	142·6	3048	72·4	97·1	2962
58·0	142·6	2781	68·9	129·4	3236
74·6	142·6	2912	70·1	129·4	3214
68·9	128·3	3135	70·8	161·7	3389
69·1	142·6	3261	66·5	129·4	2908
62·1	156·8	3030	66·7	137·5	3063
68·7	128·3	3139	71·2	129·4	2956
65·4	142·6	2996	72·4	145·6	3023
70·4	128·3	3248	69·3	129·4	3001
69·1	142·6	3117	67·4	145·6	2841
63·7	142·6	2891	69·6	161·7	3117
62·1	114·1	2667	66·2	121·3	2733
73·5	142·6	3403	74·5	121·3	2808
61·3	129·4	2999	67·7	97·1	2813
70·1	137·5	3318	57·5	97·1	2615
79·8	121·3	2989	70·4	113·2	2814
61·3	129·4	3936			

The statistical constants are shown in Table IV, and the deduced equation is

$$H = 6{\cdot}244W + 17{\cdot}777M + 977{\cdot}5. \tag{6}$$

Table IV.—Constants Deduced from Table III.

	Heat production.	Work.	Mass.
Mean	3015·70	132·392	68·157
Standard deviation	197·927	15·968	4·917

$r_{HW} = 0{\cdot}5469$

$r_{HM} = 0{\cdot}4909$

$r_{WM} = 0{\cdot}0977$

${}_{M}r_{HW} = 0{\cdot}5754$

${}_{W}r_{HM} = 0{\cdot}5250$

${}_{H}r_{WM} = -0{\cdot}2340$

${}_{WM}\Sigma_{H} = 141{\cdot}039$

${}_{HM}\Sigma_{W} = 12{\cdot}997$

${}_{HW}\Sigma_{M} = 4{\cdot}165.$

Table V compares the computed with the observed values. The agreement between the computed and the observed values is reasonably close, while, as seen in Table VI, the distribution of deviations, expressed in terms of the standard deviation of the prediction, is not an improbable one. The data just analysed differ in kind from those of Macdonald, since there is but one observation with respect to each individual, instead of a number of observations upon each of a few individuals. Again, the heat production is that of the whole day, so that the estimated efficiency is a mere average. But such a sample is of interest from the industrial standpoint, in so far as it throws light upon the question of the ration needed for varying amounts of

Table V.

Observed heat production.	Calculated heat production by equation (6).	Difference.	$\dfrac{\text{Difference}}{{}_{\mathrm{WM}}\Sigma_{H}}$.	Difference as percentage of observed value.
3398	3311	− 87	−0·62	2·6
2988	2957	− 31	−0·22	1·0
3048	3105	+ 57	+0·40	1·9
2781	2899	+118	+0·84	4·2
2912	3194	+282	+1·999	9·7
3135	3003	−132	−0·94	4·2
3261	3096	−165	−1·17	5·1
3030	3060	+ 30	+0·21	1·0
3139	3000	−139	−0·99	4·4
2996	3030	+ 34	+0·24	1·1
3248	3030	−218	−1·55	6·7
3117	3093	− 24	−0·17	0·8
2891	3000	+109	+0·77	3·8
2667	2794	+127	+0·90	4·8
3403	3174	−229	−1·62	6·7
2999	2875	−124	−0·87	4·1
3318	3082	−236	−1·67	7·1
2989	3153	+164	+1·16	5·5
2936	2875	− 61	−0·43	2·1
3020	2988	− 32	−0·23	1·1
2812	2858	+ 46	+0·33	1·6
2962	2871	− 91	−0·65	3·1
3236	3010	−226	−1·60	7·0
3214	3032	−182	−1·29	5·7
3389	3246	−143	−1·01	4·2
2908	2968	+ 60	+0·43	2·1
3063	3022	− 41	−0·29	1·3
2956	3051	+ 95	+0·67	3·2
3023	3174	+151	+1·07	5·0
3001	3017	+ 16	+0·11	0·5
2841	3085	+244	+1·73	8·6
3117	3224	+107	+0·76	3·4
2733	2912	+179	+1·27	6·5
2808	3059	+251	+1·78	8·9
2813	2787	− 26	−0·18	0·9
2615	2606	− 9	−0·06	0·3
2814	2936	+122	+0·87	4·3

Table VI.—Distribution of Deviations.

	Observed.	On the basis of a "normal" curve.
Greater than the standard deviation, positive	6	5·87
Between 0 and " " "	12	12·63
" " " " negative	12	12 63
Greater than " " "	7	5·87

muscular work. The particular work here chosen was abnormal in kind rather than in degree. In his recent treatise,* Amar has estimated the work of a metal filer in an $8\frac{1}{2}$ hours' day to be 61,600 kilogramme-metres; that of a smith as rather more than twice as much. Taking these quantities as equivalent to a medium and a heavy day's work, it will be interesting to compute from equation (6) the requisite energy requirements of a man weighing 67 kgrm. Taking the round numbers 60,000 and 120,000 kilogramme-metres, and reducing to thermal equivalents, we reach 3049 calories for the medium and 3929 calories for the heavy work.

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* Amar, 'Le Moteur Humain,' p. 527 and p. 547, Paris, 1914.

These figures are not very different from the usual standards. So far I have merely examined the data from the same standpoint as that of Glazebrook and Dye, viz., that of finding a formula which shall summarise particular experimental data, and it appears that this comparatively humble task can be readily accomplished, *i.e.*, we can, with very little arithmetical work, construct interpolation formulæ which reproduce the observations sufficiently well for the formulæ to be used to approximate to intermediate values within the range of the data. It is, however, more than doubtful whether such formulæ throw light upon the efficiency of the body as an energy transformer when this term is strictly used, and I now propose to examine the fundamental problem.

The customary method of determining the efficiency of the human body as a machine, prior to the publication of Macdonald's paper, was to subtract from the total heat production associated with the performance of a measured amount of external work the heat production of the same subject when "at rest"; the heat equivalent of the work divided by this has been termed the "net efficiency," the "gross efficiency" being the quotient of work by total heat production. Clearly the "net efficiency" will vary with the base line chosen, *i.e.*, whether "at rest" is taken to be lying on a couch, sitting still on the ergometer, rotating its pedals without load, etc. Benedict and Cathcart* discuss the choice of base line at length in their admirable monograph, and point out the inconvenient diversity of base lines used or proposed. But even these writers do not remark that the method is fundamentally open to attack. Thus, if heat transformed be a function of work, the proper measure of extrinsic efficiency would seem to be the reciprocal of the first derivative of H with respect to W, the limiting value of the incremental change of heat with work. This measure will differ considerably in value from the "net efficiency" and may even lead to a different interpretation of the results. For instance, Benedict and Cathcart† note a tendency towards an increase in both gross and net efficiencies with increasing loads. With

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* Benedict and Cathcart, 'Muscular Work: a Metabolic Study, with Special Reference to the Efficiency of the Human Body as a Machine,' Washington, 1913.

† *Op. cit.*, p. 125.

0·5 ampère load their professional subject had an efficiency of between 15 and 18 per cent. (revolution rate between 67 and 72), which rose to 24 per cent. when the load was furnished by a 1·5 ampère current. The reader might infer that the muscular efficiency improved with load. This, however, is a mere consequence of the method of calculation. Table VII gives the observations, and it will be seen that for this range the linear equation $H = aW + c$ very fairly represents the data. Adopting the above definition, the efficiency is $1/a$, in the particular case, 0·2993. Now Benedict and Cathcart's subject's net efficiency is $W/(aW + c - 1{\cdot}17)$ and his gross efficiency $W/(aW + c)$, which obviously increase with W and will only reach the value $1/a$ when W is infinite. In effect, using the formula we have, for the particular values 0·48 cal. and 1·57 cal., "net efficiencies" 0·17 and 0·24; for "gross efficiencies," 0·12 and 0·21, which agree, to this number of decimals, with the observed means.

Table VII.—Heat Production and Work. Observations of Benedict and Cathcart on M. A. M. pedalling at the rate of 68–72 revolutions per minute (inclusive).

Work done.	Observed heat production.	Mean of observations.	Heat production given by formula Heat = 3·3415 Work + 2·4131.
calories.	calories.		
0·47	4·01, 4·01	4·01	3·98
0·48	3·86, 3·88, 3·83, 4·13, 3·96	3·93	4·02
0·49	4·15, 3·94	4·05	4·05
1·19	6·73	6·73	6·39
1·20	6·65, 6·81	6·73	6·42
1·33	6·95	6·95	6·86
1·35	6·97	6·97	6·92
1·36	7·08	7·08	6·96
1·56	7·44, 7·66	7·55	7·62
1·57	7·87, 7·59, 7·53, 7·64, 7·41, 7·51, 7·55	7·59	7·66
1·59	7·56	7·56	7·73

Hence the data are quite consistent with a constant efficiency as just defined for all loads within the observed range. *A fortiori*, Amar's base line is highly objectionable, and no doubt over-estimates the real efficiency of the machine. A labourer who rests on Sunday and does eight hours' work on Monday really performs some muscular work, although not industrial work, on Sunday; the Monday's work is not a mere addition to that of Sunday, but, in part, a substitution of purposeful for aimless activity. Hence to deduct the whole of the heat production on the rest day from the working day's quota is to rate the efficiency too high.*

Macdonald's recent paper is an attempt to transfer the inquiry to a different plane altogether. If I have followed his extremely ingenious reasoning correctly, his position is as follows. He holds that the efficiency of work performance is a function of body mass, and he found that when the thermal equivalent of the external work performed was divided by the efficiency (as defined above) and the quotient subtracted from the total heat production, the residue was, in his experiments, sensibly constant for a given range of work performance. This residue is defined as the cost of movement; it varies with the velocity of movement and can again be expressed as a function of mass and rate of movement.

The details of Macdonald's analysis are perhaps open to criticism. Thus the agreement between his arithmetical calculations and the observed results is not always of such importance as appears on the face of the figures, since he is using formulæ with several constants for absolutely few observations.

Thus, he has a theoretical formula for cost of movement (Q) in terms of velocity of movement, an expression of the form $Q = a(bV)^{cV}$, where a, b and c are constants to be determined from five observations only. But these criticisms are of subsidiary importance and detract in no way from the suggestiveness of the argument. Looking at the theory as a whole, the following conclusions appear to follow. The real efficiency of the muscular machine is indeterminate and may even be unity. The apparent efficiency of the same individual performing the same kind of work at different rates will be represented by the reciprocal of the first derivative with respect to W of $H = aW + b$, *i.e.*, is equal to $1/a$; a is constant and $1/a$ might be called the indicated efficiency, but b (the cost of movement) will vary with the rate of performance, being a minimum at the most economical rate of movement. Thus the locus of heat produced as a function of work is a family of parallel straight lines intersecting the axis of heat at different points.†

It seemed desirable to test the deduction on the basis of Benedict and Cathcart's important data. To this end the observations upon their professional subject were sorted out into groups for each of which the rate of pedalling fell within narrow limits. Only two such groups were really suitable for the purpose; that already given in which the rate of pedalling was between 68 and 72 and the group 98 to 102. In the remainder either the number of observations was less than 20 or the range of variation of work performed very small. In Table VIII are reproduced the second available set. The linear equations are

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* This criticism is implicit in Lefèvre's excellent discussion of the efficiency problem ('Chaleur Animale et Bioénergétique,' pp. 924, etc., Paris, 1911), which I had not seen until most of this paper had been written.

† It is to be noted that Chauveau's formula also includes a velocity form, see Lefèvre, *op. cit.*, pp. 686 *et seq.*

$$H = 3{\cdot}3415\,W + 3{\cdot}4131 \qquad \text{and} \qquad H = 3{\cdot}61225\,W + 3{\cdot}7543$$

Table VIII.—Heat Production and Work. Observations of Benedict and Cathcart on M. A. M. pedalling at the rate of 98–102 revolutions per minute (inclusive).

Work done.	Observed heat production.	Mean of observations.	Heat production given by formula Heat = 3·61225 Work + 3·7543.
calories.	calories.		
0·55	5·59, 5·28, 6·03, 5·60, 5·64	5·63	5·74
0·56	5·72	5·72	5·78
1·35	8·72	8·72	8·63
1·62	10·15	10·15	9·61
1·63	10·88	10·88	9·64
2·05	11·24	11·24	11·16
2·06	11·30	11·30	11·20
2·07	10·82, 11·21, 11·43, 10·91	11·09	11·23
2·08	11·19	11·19	11·27
2·09	10·98, 11·65, 11·74	11·46	11·30
2·10	11·26	11·26	11·34
2·11	11·13, 11·15, 11·39, 11·35	11·26	11·38
2·12	11·29, 11·35, 11·03, 11·29	11·24	11·41

The first point which arises is whether the observations are, when examined strictly, reasonably probable examples of a linear law (that they are sufficiently so for ordinary purposes of numerical interpolation is no doubt evident). To test the matter, I had recourse to a method proposed by Pearson,[*] the application of which is seen in Tables IX and X. On the face of the figures, it seems that both cases are very improbable examples of a

Table IX.—Observations of Table VII Grouped for Testing Fit.

Group.	Frequency.	Observed mean.	Calculated mean.	Square of difference.	Square of difference × frequency.
0·47–0·49	9	3·974	4·018	0·0019	0·0172
1·19–1·33	4	6·785	6·523	0·0689	0·2756
1·35–1·56	4	7·288	7·280	0·00006	0·0002
1·57	8	7·583	7·669	0·0074	0·0595
					0·3525

$\sigma_H{}^2(1-\eta^2) = 0{\cdot}01698$; $\chi^2 = 20{\cdot}760$; $P = 0{\cdot}00036$.

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[*] 'Biometrika,' vol. 11, p. 237 (1916).

Table X.—Observations of Table VIII Grouped for Testing Fit.

Group.	Frequency.	Observed mean.	Calculated mean.	Square of difference.	Square of difference × frequency.
0·55	6	5·643	5·747	0·0108	0·0648
1·35	1	8·720	8·631	0·0079	0·0079
1·62–1·63	2	10·515	9·625	0·7921	1·5842
2·05–2·10	11	11·248	11·254	0·00004	0·0004
2·11	4	11·255	11·376	0·0146	0·0584
2·12	4	11·240	11·412	0·0296	0·1184
					1·8341

$\sigma_H{}^2(1-\eta^2) = 0·053366$; $\chi^2 = 34·368$; P = less than 10^{-4}.

linear law, but some reservations must be made. Thus in each the value of χ^2 is chiefly due to a single group, while, careful as are the experiments, absolute uniformity of external conditions can never be secured in inquiries of this kind, so that too much stress ought not to be put upon absolutely small deviations. I am not confident therefore that the result of this test is sufficient to condemn the law as being inapplicable. I also think that no significance attaches to the difference between the regression coefficients (the reciprocals of the efficiencies). The difference is $0·2708 \pm 0·0855$, which is more than three times the probable difference, but the applicability of the normal law of error to such cases may justly be questioned; that the divergence may be of no practical importance is suggested by the results obtained with the next higher rates of pedalling, although these are determined from smaller numbers of observations. They are shown in Tables XI and XII, and it will be seen that the regression coefficients do not continue to increase, but are well within the range of likely fluctuation about the value deduced from the observations at lower speeds.

Table XI.—Heat Production and Work. Observations of Benedict and Cathcart on M. A. M. pedalling at the rate of 103–107 revolutions per minute (inclusive).

Work done.	Observed heat production.	Mean of observations.	Heat production given by formula Heat = 3·1262 Work + 5·1525.
calories.	calories.		
0·55	6·01	6·01	6·87
1·35	9·20, 9·78	9·49	9·37
1·64–1·65	11·21, 11·35	11·28	10·30
1·83	11·53	11·53	10·87
2·12–2·17	11·55, 11·63, 11·72, 12·66, 11·33 11·32, 11·83, 11·73, 11·71, 11·23	11·67	11·87

Table XII.—Heat Production and Work. Observations of Benedict and Cathcart on M. A. M. pedalling at the rate of 108–112 revolutions per minute (inclusive).

Work done.	Observed heat production.	Mean of observations.	Heat production given by formula Heat = 3·3271 Work + 5·1980.
calories.	calories.		
0·54–0·55	6·46, 6·70	6·58	7·02
1·37	9·10	9·10	9·76
1·58–1·68	11·29, 11·35, 12·85	11·83	10·64
2·18–2·24	11·77, 13·02, 11·97, 12·72, 12·16 13·08, 11·59, 12·47, 12·17	12·33	12·55

I conclude, then, that the data of Benedict and Cathcart, much the most extensive at our disposal, are consistent with the hypothesis that heat production is, at least to a first approximation, expressible as a linear function of work, $H = aW + b$, a being constant for all observed ranges of work.

A matter which calls for discussion is the economy of thermogenesis in work, a question first, I think, raised by Lefèvre in 1902, and again by Lapicque in 1906.* The discussion of these writers is, although not affected in principle, complicated in detail by their adoption of the glucositic theory of muscular energy, and I shall follow a somewhat different line of thought. The fundamental notion is, of course, quite simple. A certain intensity of pure thermogenesis is necessary for the bioplasm to act as an energy transformer at all; the lower limit of this is given by the heat output at rest. But when the bioplasm performs work, there is, unless the conversion of potential energy into work is complete, an associated liberation of energy as heat and not otherwise available. Hence there is a possibility that some part of the heat production necessary for the existence of the bioplasm might be obtained, so to speak, as a by-product of muscular work. Carrying the point to an extreme, we might have, for a certain range of work performance, the range varying inversely as the real efficiency, merely to introduce equal increments of heat for each increment of work, so that within that range the apparent incremental efficiency, *i.e.*, the indicated efficiency, would be unity, whatever the value of the real efficiency. Of course, in this extreme form, the hypothesis could not possibly be true, for it would involve a complete constancy of heat production (after subtracting from the total heat production the thermal equivalent of the work performed), which, since in work performance the opportunity for heat loss must usually increase, could not happen. But the original idea might, nevertheless, be correct.

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* Lefèvre, *op. cit.*, pp. 910, etc.

The argument can best be expressed symbolically.

Let h be the resting heat production in unit of time, W the work performed (in thermal units), E′ the real efficiency, *i.e.*, W/E, the energy needed to develop W units of work, and $H' = H - h$. Then we have

$$H' = W/E' - \phi(W)h$$

subject to the conditions

$$\text{(i)}\ \phi(W) \text{ cannot exceed 1 and } \phi(0) = 0,$$

$$\text{(ii)}\ (1-E')(W/E' - \phi(W)h) + h[(1-\phi(W)] = h+\lambda,$$

where λ is an unassigned function of W.

These give
$$\phi(W) = \frac{(1-E')W/E' - \lambda}{(2-E')h},$$

and $\phi(W)$ attains its greatest possible value when

$$W = \frac{E'}{1-E'}\{2-E')h+\lambda\}.$$

So that
$$H = W/E' + h\left\{1 - \frac{(1-E')W/E' - \lambda}{(2-E')h}\right\},$$

until
$$W = \frac{E'}{1-E'}\{(2-E')h+\lambda\},$$

and thereafter
$$H = \frac{W}{E'},$$

hence
$$\frac{1}{E} = \frac{dH}{dW} = \frac{1}{E'} - \frac{(1-E')/E' - d\lambda/dW}{(2-E')} \quad \text{at first,}$$

and subsequently
$$= \frac{1}{E'}.$$

This is, however, inconsistent with the experimental facts, viz., that when W is large, although H is approximately linear in W, it is not equal to aW but to $a\text{W}+b$, and arbitrarily assumes λ to be given by $\phi(W)$. Hence we must write our equation in the more general form :—

$$H = W/E' + [1-\phi(W)]h + f(W)$$

subject to
$$\text{(i)}\ \phi(0) = f(0) = 0,$$

$$\text{(ii)}\ W[(1-E')/E'] + [1-\phi(W)]h + f(W) \nless h,$$

$$\text{(iii) At and after } W = k,$$

$$H = W/E' + b, \text{ where } b \text{ is constant.}$$

From (ii) we have
$$\phi(W) \ngtr \frac{f(W)}{h} + \frac{W(1-E')}{E'h},$$

from (iii)
$$h(1-\phi(k)) + f(k) = b,$$

hence, up to $W = k$,

$$\frac{1}{E} = \frac{dH}{dW} = \frac{1}{E'} + \frac{df(W)}{dW} - \frac{d\phi(W)}{dW} \cdot h,$$

and consequently E might differ considerably from E′, for values of W not exceeding k. But thereafter it will become constant and equal to E′, for $df(W)/dW$ and $d\phi(W)/dW$ will vanish.

The argument may be illustrated on Benedict and Cathcart's cyclist working at rates of 68–72 revolutions per minute. Here, for $W = 0{\cdot}48$ or greater, $H = 3{\cdot}3415\,W + 2{\cdot}4131$. Let us assume that constancy is reached at $W = 0{\cdot}45$, which will be the k of our formula. Hence $E' = 0{\cdot}29926$; putting $h = 1{\cdot}17$ (its mean value in Benedict and Cathcart's series), we have:—

$$1{\cdot}17\,[1 - \phi(0{\cdot}45)] + f(0{\cdot}45) = 2{\cdot}4131.$$

We have no knowledge at all of $\phi(W)$ and $f(W)$ save the initial conditions of each and the terminal value of their sum. Assuming that $\phi(0{\cdot}45) = 1$, and that each is linear for that range, we should have

$\phi(W)$ increasing by 0·222 for each 0·1 calorie for 0 to 0·45 and

$f(W)$ increasing by 0·836.

Hence for values of W up to 0·45, $E = 0{\cdot}1639$; for greater values of W it is equal to 0·2993. The locus of H is formed by two straight lines intersecting at $W = 0{\cdot}45$. Naturally the postulated forms of $\phi(W)$ and $f(W)$ are merely illustrative of the way in which the ultimate approximately linear relation might arise. Generally, the hypothesis is that

$$H = W/E' + F(W),$$

subject to $F(0) = h$ and $F(\infty) = 0$ and the practical condition that $dF(W)/dW$ shall be very small for W greater than a constant value k, say.

There is some formal analogy between this expression and the well-known equation of Chauveau,[*] and the point arises as to the sense to be attributed to the value $W = 0$.

It is evident that the proposed equation is not even now complete, because the assumption of $E' =$ a constant ignores the non-compensated element of a real thermodynamic transformation (*i.e.*, if in such an incompletely reversible cycle, H′ is the heat of the transformation, and T the absolute temperature, $\int \frac{dH'}{T}$ is not zero). It is, however, convenient to examine the case on the assumption that $W = 0$ corresponds to the point where no external work is being done nor is energy expended in maintaining potential, which would be the case, for instance, when standing erect.

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[*] See Lefèvre, *op. cit.*, p. 728.

In conclusion, I note certain physiological factors which, coming into play at the commencement of muscular work, can hardly increase in intensity *pari passu* with the output of work, but increase the loss of heat, and therefore depress the apparent efficiency.

The first is the circulatory readjustment, leading to an enhanced flow of blood through the muscles. Evidently this partial diversion of the stream through the viscera has an upper limit which may be reached at a comparatively low rate of muscular output. No measurements suitable for the present inquiry have been made.

A second factor is the loss of heat due to air currents. In such work as that on a bicycle ergometer, currents of some magnitude will be set up by the rotatory movements of the legs. Hill, Griffith and Flack* concluded that the total heat loss from the surface of a wet katathermometer was represented by:—

$$H = (a+bV^{1/2})T+(c+dV^{1/2})(F-f)^{4/3}$$

where V is the velocity of air movement, while T, a, b, c, d, F and f are independent of V. The movement factor might help to explain the rise of heat production associated with a rotation of the pedals of the bicycle ergometer by the motor, the subject performing no work, as compared with the man's heat production while sitting still on the machine.

Benedict and Cathcart observed outputs of from 2·22 calories per minute to 4·61 (rates of rotation from 58 to 104) under these conditions. The range for sitting still was from 1·35 to 1·82. This cannot, however, be the whole explanation. The rate of heat loss due to this cause must diminish with V, since, from Hill, Griffith and Flack's equation we have $dH/dV = k/V^{\frac{1}{2}}$, where k is a constant, while Benedict and Cathcart's data show that the rate of increase of heat loss is greater between 86 and 96 revolutions per minute than between 60 and 86. The observations are not, however, very numerous, and I hope to be able to submit in a subsequent paper further experimental results dealing with the point.

The results which emerge from the study here published and seem to me interesting are:—

(1) Within fairly wide ranges, simple formulæ of linear regression describe the relations subsisting between heat production, body mass and work performance with an accuracy sufficient for such purposes as roughly computing the energetic needs of workers, doing the kind of work studied. As, however, this work is of a specially simple kind, the type of calculation is

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* 'Phil. Trans.,' B, vol. 207, pp. 183–220 (1916).

more likely to be useful in connection with military exercises than if applied to industrial labour.

(2) No law connecting heat production and work performance can be properly formulated until the range of experimental observations under uniform conditions has been carried below the starting point of existing records. It is desirable to define muscular efficiency more strictly than has been usual, and it is possible that the results of Macdonald which the present extended analysis of Benedict and Cathcart's data verifies, are open to more than one interpretation.

The Galvanometric Measurement of " Emotive " Physiological Changes.

By A. D. WALLER, M.D., F.R.S.

(Received October 26, 1917.)

Object.—The object of these experiments was to determine whether or no in the absence of the ordinary visible signs of emotion (muscular, secretory, etc.), electrical signs of emotive discharges are demonstrable by galvanometer.

Affirmative results reported by previous observers* have not, to my mind, fully established the reality of the ground fact, independently of the slight and ordinarily insensible muscular movements that can be perceived by a thought reader or recorded by suitable apparatus, and, as a first step in the inquiry, I thought it necessary to take simultaneous records of galvanometric and muscular movements.

The following communication deals only with the large and sudden electrical responses that are unmistakeably independent of muscular contraction. The smaller and more gradual fluctuations of more debatable nature will be dealt with in a future communication. For the present state of the subject it is, in my opinion, necessary, in the first instance, to establish as clearly as possible the chief actual facts by actual demonstration.

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* Veraguth, 'Das Psychogalvanische Reflexphenomen," Berlin, 1909. Petersen and Jung, 'Psychophysical Investigations with the Galvanometer and Pneumograph in Normal and Insane Individuals.' Goldsheider, "Der sogenannte psycho-galvanische Reflex und seine physikalischchemische Deutung," 'Pflüger's Archiv,' vol. 162, p. 489 (1915).